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Book Review: *Thermodynamics of Chaotic Systems*

Thermodynamics of Chaotic Systems: An Introduction. Christian Beck and Friedrich Schlogl, Cambridge University Press, Cambridge, 1993.

While reading this book I was frequently reminded of Stan Ulam's statement that he was interested in the "analogies between analogies." Readers who share Ulam's interest will enjoy this book, as will many readers with more pedestrian interests. The subject of Beck and Schlogl's monograph is the use of thermodynamic analogies to quantify and understand chaotic behavior. The book is designed as an elementary introduction to this set of analogies that is "accessible to a broad spectrum of the physics community," an ambitious undertaking, given the forbidding primary literature on this subject.

The potential usefulness of thermodynamic analogies for study of chaotic systems is easily illustrated with a simple comparison. In a macroscopic object, the thermodynamic quantities one measures are macroscopic parameters that replace the unimaginable complexity of the full dynamics of an Avogadro's number of microscopic components. Thermodynamic formalism relates parameters obtained in different macroscopic measurements, while statistical mechanics provides the bridge between microscopic dynamics and macroscopic properties. In a number of relatively simple chaotic systems one already has a detailed description of the complex dynamical properties of the system. A knowledge of these dynamical properties is analogous to a knowledge of all of the trajectories of all of the atoms in a macroscopic sample of gas, in that this complete description of the system is too complex and too information-intensive for human understanding. Therefore, a quantitative, reduced description of chaotic dynamics analogous to thermodynamics is desirable. The present book describes how such an analogous description has been developed. As in conventional thermodynamics, a bridge between detailed dynamics and reduced description is phrased in terms of a probabilistic description. Specifically, the quasistochastic nature of chaotic motion is used to define several probability distributions over the phase space of the motion. In

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contrast with conventional statistical mechanics, there are several different probability distributions relevant to chaotic dynamics and, outside of some special systems, these distributions are not related. Each distribution describes a different property of the dynamics. Through an artifice called the escort distribution, each probability distribution generates a family of related distributions parametrized by a real number which corresponds to an inverse temperature in the formalism. Thus, each probability distribution in turn generates a family of escort distributions, which generates a thermodynamic description. The resulting thermodynamic formalism provides the desired reduced description of the system, and also can do more by way of characterizing the dynamical system. Remarkably, important measures of chaotic dynamics appear naturally as thermodynamic potentials in the different formalisms. For example, the Hausdorff dimension, Shannon information, Kolmogorov–Sinai entropy, and Liapunov exponents are all easily related to different definitions of the free energy. Furthermore, the thermodynamic formalisms provide means for generalizing and manipulating these important quantities.

The book is divided into five major parts, the first two being brief reviews. I found these reviews useful for setting the stage for the main argument of the book and for placing the following subject matter in the larger context of dynamical systems and statistical mechanics. The first part briefly reviews nonlinear dynamics, emphasizing probabilistic descriptions of dynamical systems and symbolic dynamics. The second part briefly reviews thermodynamics and information theory, emphasizing the connections between these two subjects. Part three begins the study of thermodynamics of chaotic systems. Here escort distributions are first defined and used to study time-independent properties of ergodic dynamical systems. The free energies defined in this part are related to fractal dimensions and the Shannon definition of information. In part four the concepts developed in the previous sections of the book are used to study the dynamical behavior of chaotic systems. The free energies defined in this part are related to the Kolmogorov–Sinai entropy and the Liapunov exponents. The final part of the book discusses two advanced topics, namely the unification of the different thermodynamic formalisms and phase transitions.

I found the book well-written and clearly organized. The authors have tried to write a pedagogical introduction, and for the most part have succeeded. The reviews in the first two parts of the book are a nice touch. They make the book accessible to a wide audience and also set the terminology and notation for the following parts. The presentation is concise in a way that brings out the basic structure of the theory. There is a clear place for an introductory book written in this style in a field like the thermodynamics of chaos, where the primary literature is mainly concerned with detail. The conciseness does, however, have some drawbacks. Reading this book requires considerable concentration as one is constantly being presented with new concepts. I also would have liked to see more treatment of numerical aspects of chaos. Overall, however, I enjoyed reading this excellent book. I can easily recommend it to any reader of the *Journal of Statistical Physics* who finds the subject matter of interest. Its pedagogical quality makes it accessible to the interested novice and its clarity of organization will make it useful to experts.

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